Multifractal Analysis of Au-Au collisions at RHIC Energies

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Introduction

The fluctuation analysis of measurable quantities such as multiplicity fluctuation at relativistic heavy-ion collider (RHIC) experiments in terms of the multifractal behavior of charged particle spectra has been proven of utmost significance. Multifractal analysis not only focuses on the production process of the high energy relativistic charged particles but also provides us a tool to understand the nature of Quantum Chromo Dynamics (QCD) phase transition. Takagi [1] proposed a new methodology to probe the multifractal structure of a multiparticle production process. The method was utilized to analyze TASSO [3] and DELPHI data [4] on e^+e^- annihilations and UA5 data [2] on $p\bar{p}$ collisions to explore the fractal behavior of the charged particle spectra. Recent years have witnessed the wide applicability of the method to extract relevant information on the nature of QCD phase transition in heavy-ion collisions at relativistic energies. This article explores the multifractal behavior of the charged particle multiplicity fluctuations in HYDJET++ generated Au-Au collision events at three RHIC energies. Furthermore, we explore the centrality dependence of the multifractal behavior at three RHIC energies and report the extracted values of the multifractal specific heat.

Methodology Used for Analysis

To investigate the non-statistical fluctuations and to infer a quantitative understanding of the multifractal properties of heavy-ion collisions at collider energies, the Takagi moment approach has been employed here to analyze HYDJET++ generated data.

If we consider a single event containing n relativistic charged particles, distributed in the pseudorapidity (η) space in the interval $\eta_{min} < \eta < \eta_{max}$. The η interval $\Delta\eta$ ($= \eta_{max} - \eta_{min}$) is divided into M bins of equal size so that $\delta\eta = (\Delta\eta/M)$. For each such bin, the multiplicity distribution of charged particles can be represented as $P_n(\delta\eta)$ for $n = 0, 1, 2, 3, \ldots$. It is assumed that the inclusive η distribution $dn/d\eta$ is constant and $P_n(\delta\eta)$ is independent of the location of the bin. The multiplicity of the charged particles varies from event to event according to the distribution $P_n(\Delta\eta)$ [1]. Let N be the total number of charged secondary particles produced in these PΩ number of independent events and n_{ij} be the multiplicity of produced charged particles in the j^{th} bin of the i^{th} event such that the normalized density P_{ij} can be defined by [1]: $P_{ij} = n_{ij}/N$ and also holds true when $N \longrightarrow \infty$. P_{ij} is directly related to the Takagi moment T_q of order q by the relation: $T_q(\delta\eta) = ln\sum_{i=1}^{\Omega}\sum_{j=1}^{M} P_{ij}^q$ for $q > 0$. The above relation behaves like a linear function of the logarithm of the resolution $R(\delta\eta)$ as given: $T_q(\delta\eta) = A_q + B_q lnR(\delta\eta)$. Here A_q and B_q are the constants independent of $\delta\eta$. This relation follows a linear behavior and is easily obtained for the simplest choice for $R(\delta\eta) = \delta\eta$. If such a behavior is observed for a considerable range of $R(\delta\eta)$, values of generalized dimension D_q are estimated as: $D_q = B_q/(q-1)$ Takagi suggested that $\langle n\rangle$ would be a better choice of the resolution $R(\delta\eta)$ because $\frac{dn}{d\eta}$ is flat by definition [1]. Choosing$R(\delta\eta) = \langle n\rangle$, we observe a simple linear relation between $ln\langle n^q\rangle$ and $ln\langle n\rangle$ as expressed by $ln\langle n^q\rangle = A_q + (B_q+1)ln\langle n\rangle$. The

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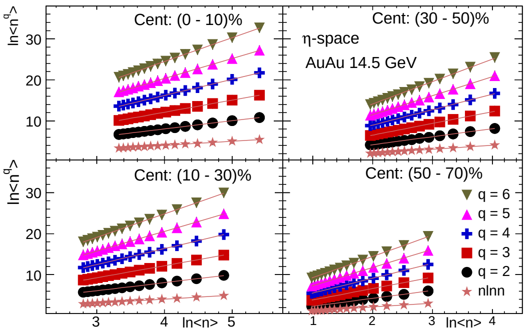

FIG. 1: Variation of $ln\langle n^q\rangle$ with $ln\langle n\rangle$ for charged particles generated for the HYDJET++ events in η-space at $\sqrt{s_{NN}}$ = 14.5 GeV.

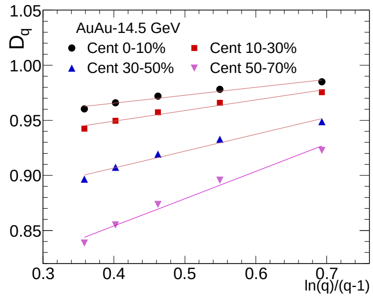

FIG. 2: Dependence of D_q on the $ln(q)/(q-1)$ for charged particles generated by the HYDJET++ events in η-space at four centrality intervals at $\sqrt{s_{NN}}$ = 14.5 GeV.

case with $q = 1$ can be realized by taking an appropriate limit. The value of information dimension D_1 can also be calculated using simple relation as suggested by Takagi [1]: $\frac{\langle nlnn\rangle}{\langle n\rangle} = C_1 + D_1 ln\langle n\rangle$ It has been suggested that in constant heat approximation, D_q, dependence on q acquires the following form: $D_q \approx (a-c) + c\frac{lnq}{q-1}$ where 'a' is the information dimension, D_1, while 'c' denotes the multifractal specific heat. The linear dependence D_q values with $ln(q)/(q-1)$, is expected to be observed for multifractals.

Results and Discussions

A multifractal analysis using Takagi moment approach is carried out in η-phase space for charged particles generated using the HYDJET++ model for Au-Au collisions at $\sqrt{s_{NN}}$ = 14.5 GeV, 62.4 GeV and 200 GeV. A total of 2×10^6 events at each energy have been generated and divided into four centralities (0-10%, 10-30%, 30-50%, and 50-70%) intervals. Fig.1 shows the variation of $ln\langle n^q\rangle$ vs $ln\langle n\rangle$ for charged particles generated for the HYDJET++ events in η-space at $\sqrt{s_{NN}}$ = 14.5 GeV, 62.4 GeV and 200 GeV. The generalized dimensions have been calculated from the best-fitted slopes to the data points in the region of linear behavior of $ln\langle n^q\rangle$ values with $ln\langle n\rangle$ for all the four centrality classes at three considered RHIC energies. In Fig.2, the dependence of D_q values on the $ln(q)/(q-1)$ for four centrality intervals at 14.5 GeV are shown. The variations of D_q against $ln(q)/(q-1)$ are found to be linear from central to peripheral collisions and hence, have

TABLE I: Values of 'c' on the different centrality intervals at different RHIC energies.

Cent (%)	14.5 GeV	62.4 GeV	200 GeV
(0-10)	0.072±0.008	0.071±0.008	0.062±0.007
(10-30)	0.097±0.010	0.095±0.010	0.084±0.009
(30-50)	0.153±0.014	0.150±0.014	0.136±0.013
(50-70)	0.248±0.020	0.243±0.018	0.227±0.017

been attempted to be fit with a linear function of 1^{st} order. The linear dependence of D_q values with $ln(q)/(q-1)$ is indicative of multifractal behaviour of the charged particle production in Au-Au collisions at 14.5, 62.4, and 200 GeV energies. The slope 'c' is referred to as the multifractal specific heat, and, hence values are obtained for different set of data listed in Table 1. It is noticed for a given centrality class the value of 'c' is observed to be nearly the same within the statistical limits. A slight deflection from the constancy might be due to centrality selection of events. These findings are thus in accord with those observed earlier in pp and AA collisions. It is remarkable to observe that the constant specific heat (CSH) approximation is applicable to the multiparticle production in high energy collisions too.

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