Anisotropy in magnetized quark matter with anomalous magnetic moment of quarks

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Introduction

The goal of relativistic heavy ion collision (HIC) experiments is to study strongly interacting matter. It is believed that very strong and transient ($\sim$ few fm/c) magnetic fields of the order $\sim 10^{15-18}$ Gauss or larger might be generated in non-central HICs due to the receding charged spectators. However, the finite electrical conductivity of the medium ($\sim$few MeV) can delay the decay process of the magnetic field. Further, the existence of anomalous magnetic moment (AMM) of quarks is a well-known phenomenon arising due to quantum fluctuations and significantly affects the properties of strongly interacting matter.

In this article, we study the anisotropy (in speed of sound and isothermal compressibility) of strongly interacting quark matter due to the presence of background magnetic field at finite temperature and chemical potential considering both with and without the AMM of quarks. The system is described within the framework of Polyakov-Nambu-Jona-Lasinio (PNJL) model. This model respects the fundamental properties of QCD such as chiral symmetry breaking, confinement-deconfinement transition and hence useful for studying the thermodynamics of quark matter, especially near the crossover between hadronic and quark-gluon plasma phases.

Formalism

A. PNJL model

The Lagrangian density of the two-flavor PNJL model in presence of background magnetic field with AMM of free quarks is [1]

$$
\begin{aligned}
\mathcal{L} &= \bar{q}(x)\left(i\gamma^{\mu}D_{\mu} - m + \gamma^{0}\mu_q + \frac{1}{2}\hat{a}\sigma^{\mu\nu}F_{\mu\nu}\right)q(x) \\
&\quad + G\left\{(\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma_5\tau q(x))^2\right\} \\
&\quad - U\left(\Phi,\bar{\Phi},T\right)
\end{aligned}
$$

Employing mean field approximation on the Lagrangian, one can obtain the thermodynamic potential for 2-flavour PNJL model using imaginary time formalism of finite temperature field theory as [1]

$$
\begin{aligned}
\Omega &= \frac{B^2}{2} + \frac{(M-m_0)^2}{4G} + U\left(\Phi,\bar{\Phi},T\right) \\
&\quad - 3\sum_{nfs}\frac{|e_fB|}{2\pi}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}\omega_{nfs}f_{\Lambda} \\
&\quad - T\sum_{nfs}\frac{|e_fB|}{2\pi}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}\left\{\ln g^{+} + \ln g^{-}\right\}
\end{aligned}
$$

where n is Landau level, $s \in (\pm 1)$ and

$$
\begin{aligned}
g^{+} &= g^{+}(\Phi,\bar{\Phi},T,\mu_q) = 1 + 3(\Phi + \bar{\Phi}e^{-\frac{\omega_{nfs}-\mu_q}{T}}) \\
&\quad e^{-\frac{\omega_{nfs}-\mu_q}{T}} + e^{-3\frac{\omega_{nfs}-\mu_q}{T}}, \\
g^{-} &= g^{-}(\Phi,\bar{\Phi},T,\mu_q) = g^{+}(\bar{\Phi},\Phi,T,-\mu_q) \quad .
\end{aligned}
$$

Here, we have used smooth cutoff regularization procedure and introduced a multiplicative form factor as the model is non-renormalizable due to point-like quark interactions as

$$
f_{\Lambda} = \frac{\Lambda^{2N}}{\Lambda^{2N} + \left\{p_z^2 + (2n+1-s)|e_fB|\right\}^{N}} \ ,
$$

where N and Λ are fixed to reproduce the vacuum values of the pion decay constant f_{π} and the pion mass m_{π}. The energy eigenvalues of the quarks is obtained as

$$
\begin{aligned}
\omega_{nfs} &= \sqrt{p_z^2 + (M_{nfs} - s\kappa_f e_f B)^2}, \\
M_{nfs} &= \sqrt{M^2 + (2n+1-s)|e_fB|}.
\end{aligned}
$$

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The constituent quark mass M and the expectation values of the Polyakov loops Φ and $\bar{\Phi}$ can be obtained self-consistently from Eq.(1) by using the stationary conditions :

$$\frac{\partial \Omega}{\partial M} = 0; \quad \frac{\partial \Omega}{\partial \Phi} = 0; \quad \frac{\partial \Omega}{\partial \bar{\Phi}} = 0.$$

Furthermore, the expressions for quark the number density (n_q), entropy density (s) and magnetization ($\mathcal{M}$) are obtained from Ω.

B. Speed of sound and isothermal compressibility

The speed of sound is crucial for understanding the hydrodynamic evolution of matter and helps to determine the appropriate trajectory in the QCD phase diagram. In the presence of a magnetic field, it becomes anisotropic because pressure varies with direction of the magnetic field. The parallel and perpendicular components of the speed of sound with respect to the magnetic field direction along isentropic curves are given by

$$c_x^{2\,(\|)}(T,\mu_q) = \frac{\left(\frac{\partial p^{\|}}{\partial T}\right)_{\mu_q}\left(\frac{\partial x}{\partial \mu_q}\right)_T - \left(\frac{\partial p^{\|}}{\partial \mu_q}\right)_T\left(\frac{\partial x}{\partial T}\right)_{\mu_q}}{\left(\frac{\partial \epsilon}{\partial T}\right)_{\mu_q}\left(\frac{\partial x}{\partial \mu_q}\right)_T - \left(\frac{\partial \epsilon}{\partial \mu_q}\right)_T\left(\frac{\partial x}{\partial T}\right)_{\mu_q}},$$

$$\begin{aligned} c_x^{2\,(\perp)}(T,\mu_q) &= c_x^{2\,(\|)} \\ &- B\frac{\left(\frac{\partial \mathcal{M}}{\partial T}\right)_{\mu_q}\left(\frac{\partial x}{\partial \mu_q}\right)_T - \left(\frac{\partial \mathcal{M}}{\partial \mu_q}\right)_T\left(\frac{\partial x}{\partial T}\right)_{\mu_q}}{\left(\frac{\partial \epsilon}{\partial T}\right)_{\mu_q}\left(\frac{\partial x}{\partial \mu_q}\right)_T - \left(\frac{\partial \epsilon}{\partial \mu_q}\right)_T\left(\frac{\partial x}{\partial T}\right)_{\mu_q}} \end{aligned}$$

where $x = s/n_q$. Isothermal compressibility is considered a sensitive quantity for indicating the fluctuation of the order parameter during a phase transition. Because of the anisotropy in pressure, the isothermal compressibility K_T splits into parallel and perpendicular components as

$$\begin{aligned} K_T^{(\|)} &= \frac{1}{n_q^2}\left(\frac{\partial n_q}{\partial \mu_q}\right)_T \quad , \\ K_T^{(\perp)} &= \frac{1}{n_q(n_q - B\left(\frac{\partial \mathcal{M}}{\partial \mu_q}\right)_T)}\left(\frac{\partial n_q}{\partial \mu_q}\right)_T . \end{aligned}$$

Numerical Results

In Figs 1(a)-(b), our study reveals that the speed of sound and isothermal compressibility become anisotropic with respect to the direction of the background magnetic field, splitting into parallel and perpendicular directions with respect to the magnetic field. In Fig. 1(a), the splitting between $c^{2(\|)}_{s/n_q}$ and $c^{2(\perp)}_{s/n_q}$ $\left(c^{2(\perp)}_{s/n_q} > c^{2(\|)}_{s/n_q}\right)$ increases with increasing magnetic field. With the inclusion of AMM of quarks, we find that the magnitude of $c^{2(\|,\perp)}_{s/n_q}$ increases. On the other hand, in Fig. 1(b), $K_T^{\|}$ decreases and $K_T^{\perp}$ increases with increasing background magnetic field. Hence, the system is less compressible along the magnetic field direction with increasing background magnetic field. Therefore, the EoS becomes more stiff along the field direction compared to other directions

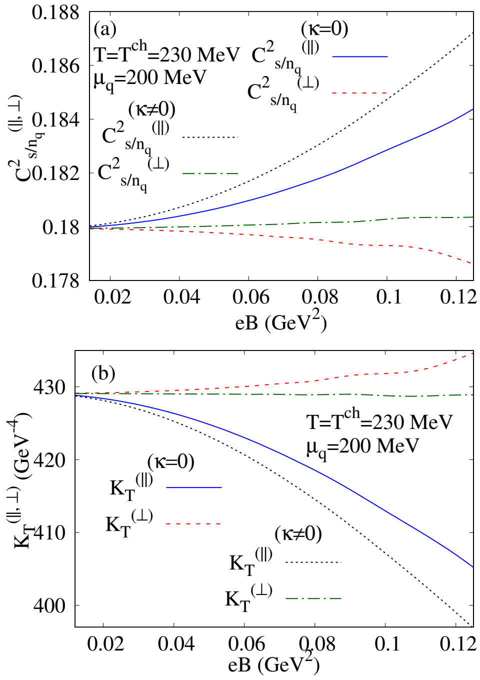

FIG. 1: (Color online) (a) $c^{2(\|,\perp)}_{s/n_q}$ and (b) $K_T^{2(\|,\perp)}$ as a function of eB. $T^{\rm ch}$ is chiral phase transition temperature when $\mu_q = 0$.

References

[1] R. Mondal, S. Duari, N. Chaudhuri, S. Sarkar, P. Roy; *Phys. Rev. D* 110, 054010.