Matter effect and CP Violation in neutrino oscillation using Quantum Computer

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Introduction

Recent years have witnessed increasing activity to develop quantum simulation techniques to understand various complex problems. Particularly in High Energy Physics, the ability of quantum computers to simulate actual Hamiltonian evolution is an advantage over classical systems. Neutrino oscillation is a phenomena where quantum superposition is observed at long distances. Thus efforts are being made to adapt this problem solvable on quantum computer.

In earlier works [1], basic Q-gate operations involved in neutrino oscillations have been presented. In this work, we present techniques to understand matter effect and CP violation using quantum simulations. Such studies can provide more efficient methodology to study oscillations in more complex systems such as Sun, Supernovae or the Nuclear stars etc.

Formalism and Theoretical aspects

In general, the probability of neutrino oscillation from one flavour, α to another, β in vacuum can be written as -

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \times \exp\Big(-\frac{i\Delta m^2_{kj} L}{E}\Big), \tag{1}$$

where, (α,β) could be anything from e,μ,τ and $(k,j) = 1,2,3$. The weight factors, $U_{\alpha k}$ are the elements of PMNS matrix.

Neutrinos that propagate through matter, experince a coherent forward elastic scattering with the electrons and the nucleons in the medium. ν_e scatter via exchange of W$^\pm$ and Z^0 with electron and by exchange of Z^0 with neucleons. ν_μ and ν_τ scatter only via Z^0 exchange. Therefore neutrinos in matter will have different effective mass over in vacuum.

We will consider the simplest case of 2 flavour oscillations in matter. The PMNS matrix is modified in this case and given by -

$$U^m = \begin{pmatrix} \cos\theta^m & \sin\theta^m \\ -\sin\theta^m & \cos\theta^m \end{pmatrix} \tag{2}$$

and the relation between mixing angle in matter, θ^m and the vacuum parameters, θ and Δm^2 is given by -

$$\cos 2\theta^m = \frac{\cos 2\theta - \beta}{\sqrt{\cos 2\theta - \beta)^2 + (\sin 2\theta)^2}}, \tag{3}$$

where $\beta = 2\sqrt{2} G_F \rho E/\Delta m^2$, G_F is the Fermi constant and ρ is the number density of the matter. Probability of neutrino flavour oscillation in matter is gven by -

$$P^m(\nu_e \to \nu_\mu) = \sin^2 2\theta^m \times \sin^2\Big[\frac{\Delta m^2 L}{4E}\Big(\sqrt{\cos 2\theta - \beta)^2 + (\sin 2\theta)^2}\Big)\Big]. \tag{4}$$

We have considered the Earth's core as a medium to show the oscillations in the matter. Δm^2 to be taken as 0.002 eV^2, E is the energy of neutrino at the time of production taken to be 1 GeV, the mixing angle in vacuum is taken to be 0.2950 rad for which the mixing angle in Earth core comes out to be 0.2986 rad. $\sqrt{2}G_F$ is 7.63$\times$10^{-14} eVcm$^3/N_A$ and ρ for Earth core is in the order of 10^{23} cm^{-3} which gives value of β to be 0.01267.

Figure 1 shows the comparison between the transition probability of ν_e to ν_μ oscillations in vacuum and in the Earth core.

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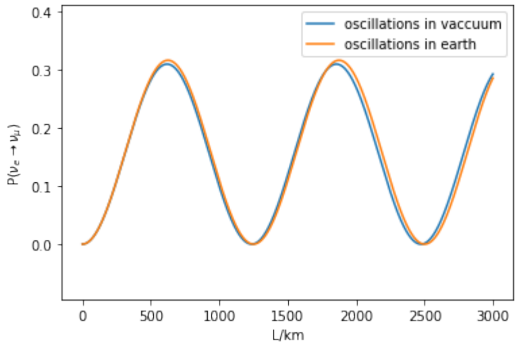

FIG. 1: Transition probability of ν_e to ν_μ oscillations in vacuum and in the Earth core.

Quantum Simulations and Results

To perform this operation in Qiskit, we have used single qubit to define two mass eigenstates, i.e. -

$$|0\rangle = |\nu_1\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad |1\rangle = |\nu_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}. \tag{5}$$

and their rotation into two flavour basis is given by -

$$|\nu_e\rangle = U^\dagger_{\text{PMNS}}\,|0\rangle \quad \text{and} \quad |\nu_\mu\rangle = \mathrm{U}^\dagger_{\text{PMNS}}\,|1\rangle \tag{6}$$

PMNS matrix in matter is given by Eq. 2.
Next, the time evolution of these mass eigenstates is represented using a P-Matrix, i.e. -

$$\begin{gathered} U(t) = P(\lambda) = \begin{pmatrix}1 & 0\\0 & e^{i\lambda}\end{pmatrix}; \\ \lambda = \frac{\Delta m^2 t}{2E\hbar} = \frac{\Delta m^2 L}{2E}\text{(in natural units)}. \end{gathered} \tag{7}$$

The Q-gate circuit for the two flavour neutrino oscillation is shown in Figure 2. Here two qubits, q_0 and q_1 are present. Three gates were being operated on q_0 for one value of λ which depends on L.

Figure 3 shows the simulated results for the oscillation in Earth core medium along with theoretical results for the same. We can see that result matches well.

Studying CP violation

We can also study the CP violation using Q-gates. For that, first we need to consider the three flavour transitions. Next we will modify the PMNS matrix accordingly by adding the phase term to it. Then looking into the difference between the probability distributions of the transitions for U_{PMNS} and $U^\dagger_{PMNS}$, we can get the CP Asymmetry, i.e.

$$A^{CP}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu_\alpha} \to \bar{\nu_\beta}} \tag{8}$$

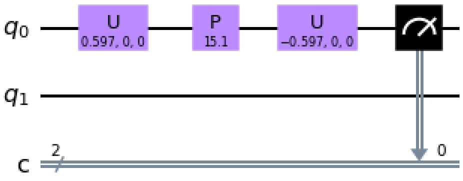

FIG. 2: Circuit diagram to calculate the probabilities of two flavour neutrino oscillations.

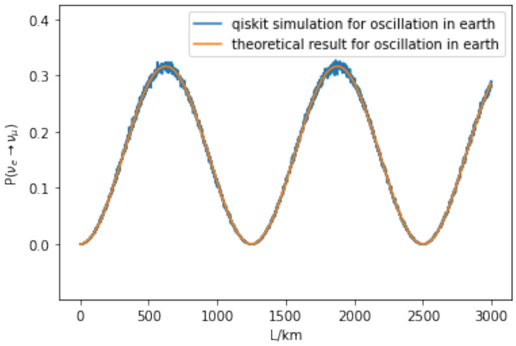

FIG. 3: Qiskit simulation and theoretical results for the transition probability of ν_e to ν_μ oscillation in the Earth core.

Summary

In summary, we have presented calculations for the Earth matter on two flavour neutrino oscillations. The calculations for the three flavour at quantum computer along with CP violation will be presented.

References

[1] Argüelles, C. A. and Jones, B. J. P., Phys. Rev. Research., **1** 2019, 033176