Simulating neutrino oscillations on a quantum circuit using virtual-Z gates

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Introduction

In quantum computing the state of a qubit can be represented as a point on the Bloch sphere. The computation is performed by different unitary gates that correspond to the rotation of the qubit state on the Bloch sphere. In particular, rotations around the Z axis, i.e., Z gates correspond to a change in the relative phase between $|0\rangle$ and $|1\rangle$. An efficient way to implement Z gates is by adjusting the reference frame concerning the qubit state. In practice this can be done by adding a phase offset to the drive field for all subsequent X and Y gates [1]. Since this does not require any physical pulse, this "virtual"-Z gate is free from calibration errors and has zero gate duration. As an interesting application, we present the usefulness of virtual-Z gates in the quantum simulation of neutrino oscillations. The main utility of virtual-Z gates lies in reducing the number of physical pulses required for circuit simulation, which improves the efficiency of the algorithm performed on noisy qubits.

The standard formalism for evaluating the neutrino oscillation probabilities involves: (*i*) changing the basis from flavor eigenstates to mass eigenstates, (*ii*) time evolution of the neutrino state in mass eigenbasis, (*iii*) converting back to the flavor eigenstates and measurement of the neutrino flavor state. Thus if neutrino is initially produced in a flavor state $|\nu_\alpha\rangle$, the oscillation probability of measuring the neutrino in flavor state $|\nu_\beta\rangle$ is given by: $P(\nu_\alpha \to \nu_\beta) = |\langle\nu_\beta| U_{\rm PMNS} U(t) U^\dagger_{\rm PMNS} |\nu_\alpha\rangle|^2$, where $U_{\rm PMNS}$ is the mixing matrix and $U(t)$ describes the evolution of the neutrino mass eigenstates.

In the case of two flavor neutrino oscillations, we encode the two neutrino flavor states in a single qubit as: $|\nu_e\rangle = |0\rangle$ and $|\nu_\mu\rangle = |1\rangle$. The quantum circuit for calculating the oscillation probability includes a phase gate sandwiched between two rotation gates: $U(-2\theta, 0, 0) R_Z(\phi) U(2\theta, 0, 0)$, where θ is the mixing angle. The phase angle is given by $\phi = \Delta m^2 L/2E$, where Δm^2 $(= 7.5 \times 10^{-5}$ $eV^2)$ is the mass squared difference, L $(= 150$ Km$)$ is the distance between source and detector and E is neutrino energy. The virtual-Z gate can be used to implement the phase gate in the following manner:

$$\begin{aligned} & U(2\theta,0,0)R_Z(\phi)U(-2\theta,0,0) \\ & = R_X(2\theta)R_Z(\phi)U(-2\theta,0,0) \\ & = R_Z(\phi)R_Z(-\phi)R_X(2\theta)R_Z(\phi)U(-2\theta,0,0) \\ & = R_Z(\phi)U(2\theta,-\phi,\phi)U(-2\theta,0,0). \end{aligned} \quad (1)$$

The additional $R_Z(\phi)$ gate at the end does not change the probabilities since the measurements are done in the Z-basis. Simulating three-flavor neutrino oscillations requires a quantum circuit with two-qubit entangling gates. We use the parametrization scheme shown in [2] which uses six single qubit rotations and two controlled-X gates to encode the

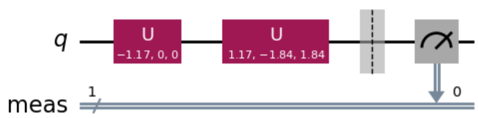

FIG. 1: Quantum circuit for simulating two-flavor neutrino oscillations using the virtual-Z gate. We perform the simulation in both *Qiskit Aer* simulator and IBM Q machine.

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PMNS matrix. The rotation angle parameters were obtained by optimizing the resultant unitary. An application of virtual-Z gates yields:

$$U_{\text{PMNS}} = \big(U^A(\alpha_1,0,0)\otimes U^B(\beta_1,0,0)\big)U_{\text{CX}}^{\text{B}\to\text{A}}\big(U^A(\alpha_2,0,0)\otimes U^B(\beta_2,0,0)\big)U_{\text{CX}}^{\text{B}\to\text{A}} \big(U^A(\alpha_3,0,0)\otimes U^B(\beta_3,0,0)\big). \tag{2}$$

$$U_{\text{PMNS}}\big(R_Z^A(\phi_2)\otimes R_Z^B(\phi_1)\big)U_{\text{PMNS}}^{\dagger} = \big(R_Z^A(\phi_2)\otimes R_Z^B(\phi_1)\big)\big(U^A(\alpha_1,-\phi_2,\phi_2)\otimes U^B(\beta_1,-\phi_1,\phi_1)\big)U_{\text{CU}}^{\text{B}\to\text{A}}(\pi,-(\phi_2+3\pi/2),\phi_2+3\pi/2,3\pi/2) \big(U^A(\alpha_2,-\phi_2,\phi_2)\otimes U^B(\beta_2,\phi_1,\phi_1)\big)U_{\text{CU}}^{\text{B}\to\text{A}}(\pi,-(\phi_2+3\pi/2),\phi_2+3\pi/2,3\pi/2) \big(U^A(\alpha_3,-\phi_2,\phi_1)\otimes U^B(\beta_3,-\phi_1,\phi_1)\big)U_{\text{PMNS}}^{\dagger}. \tag{3}$$

Results and Discussion

In Figures 2 and 3, we plot the neutrino oscillation probabilities for two-flavor and three-flavor cases respectively. The quantum simulations were performed using the *Qiskit Aer* simulator (version 0.15.0) and the IBM Q (*ibm_sherbrooke*) platform. The results obtained using the quantum circuit agree nicely with the theoretical curves over a wide range of L/E values.

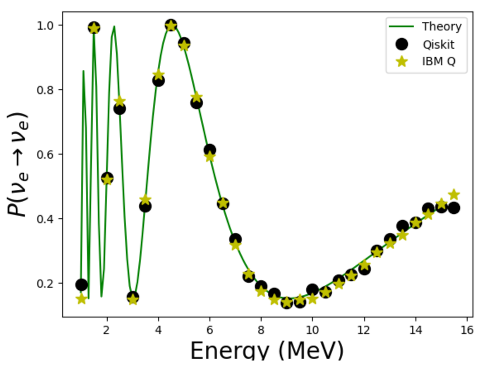

FIG. 2: Two-flavor survival probability for ν_e. The calculations with theory, *Qiskit Aer* simulator and the IBM Q machine are shown.

To summarize, we presented the study of 3-flavor neutrino oscillations using quantum circuits with virtual-Z gates and compared the results with the theory. We can use these preliminary findings to study more complicated cases involving matter effects and $CP-$violations.

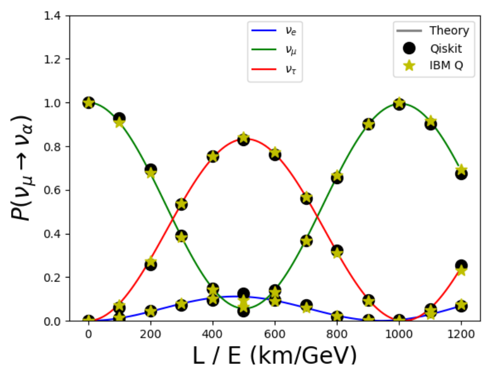

FIG. 3: Three-flavor neutrino oscillation probabilities obtained using a quantum circuit with virtual-Z gates.

References

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[2] C.A. Argüelles and B.J.P. Jones, Phys.Rev.Res. **1**, 033176 (2019).