Designing of resonator for ion trapping.

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Introduction

A charged particle confined in a Penning trap exhibits a complex motion consisting of three eigenmotions [1]. The frequencies corresponding to these eigenmotions can be measured with very high precision in a Penning trap[1]. This precise measurement allows for the highly accurate determination of the masses of trapped charged particles. Such mass measurements have significant implications, including the search for long-lived isomeric states [2], the measurement of nuclear halo charge radii[3], and the study of rapid shape transitions[4] etc.

The motion of charged particles is typically detected using resonant detection methods, which require a resonator with high impedance at the probing frequency. In the confined space of a cryogenic Penning trap setup, a helical resonator is often the preferred choice, as it provides a reasonably high quality factor within a compact geometry. This type of resonator functions as a transmission line resonator, where its characteristic dimensions are comparable to the electromagnetic (EM) wavelength. For a low-frequency resonator operating around the 900 kHz range, the wire length is approximately 50-60 meters [5]. Given the spatial constraints and the need for field confinement, the helical inner coil can be configured in a toroidal shape. This paper discusses the design of a 900 kHz toroidal resonator and includes calculations for the effective capacitance and inductance of the resonator.

Design Framework

In an ideal Penning trap, the axial oscillation frequency of the trapped particle can be written as $\omega_z = \sqrt{\frac{qU}{md^2}}$. Here U is the depth of the trapping potential, d is the characteristic dimension of the trap, and the other symbols have their usual meanings. For the VECC Penning trap, the axial frequency is approximately 40 MHz for electrons, and a resonator with unloaded frequency of 95 MHz [6] was used in detection electronics. For trapping protons, helium, and ^{7}Be ions, this frequency ranges from about 400 to 600 kHz. Taking into account the effects of the loaded trap capacitance and the connecting cable capacitance, a resonator of around 1 MHz is needed. However, within a certain range, the frequency can be fine-tuned by adjusting the depth of the trapping potential and the load capacitance

According to the well-established empirical relationship [7] for quarter-wave helical resonators of diameter D with copper conductors, the resonance frequency (f_0) and quality factor (Q) generally follow the relationship of $Q = 50Df_0^{1/2}$. Therefore, if the resonance frequency decreases from 100 MHz to 1 MHz, the Q factor will drop significantly, making it unsuitable for trapping experiments. To address this, such resonators are typically made from superconducting materials, with NbTi (niobium-titanium) being preferred due to its robustness in high-magnetic field applications and its ductility [5]. For the high-frequency resonator, we have an existing COMSOL simulation model that closely matches the experimental results within the margin of error [6]. The simulation has shown that increasing the electrical conductivity of the coil material significantly boosts the Q factor. However, a similar increase in the Q factor was not observed for changing the conductivity of the shield material. Based on these findings, we chose copper for the shield material and NbTi (niobium-titanium) for the coil material. This choice is advantageous for Penning trap experiments because a larger volume of NbTi in the shield could disrupt the homogeneity of the magnetic field [5].

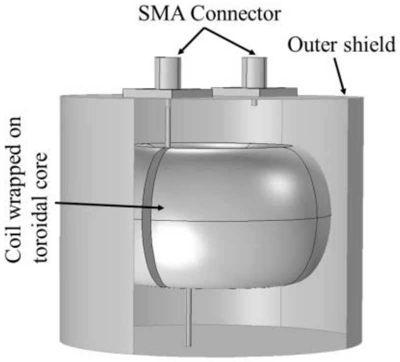

Figure1: Schematic of typical toroidal resonator. (Dimension are not to scale)

While multifilament NbTi wire with a copper matrix is generally preferred for its uniform cryogenic cooling and flux pinning properties, bare NbTi filament is preferred for alternating field applications to minimize eddy current heating. The existing COMSOL simulation in the radio frequency package cannot be directly applied because of the large number of coil turns (approximately 600-700) and the nearly zero pitch of the winding. Therefore, we used a combination of empirical methods and COMSOL simulations with the AC/DC package to determine the impedance of the resonator.

In Reference [5], a toroidal resonator with a frequency of 841 kHz was constructed using a wire length of approximately 56 meters wound around a toroidal coil with an outer diameter of about 51 mm. Although such a resonator can be configured in a helical coil arrangement [8], the magnetic flux would remain confined within the toroid, minimizing its coupling with the shield. However, in our setup, the shield diameter cannot exceed 50 mm. Therefore, we chose to wind the coil around a PTFE toroidal core with a mean radius of 13 mm. The cross-section of the toroid is elliptical, with major and minor axes of 20 mm and 5 mm, respectively, as shown in Fig 2. This configuration results in approximately 654 turns for our winding. The core is split into two semi-toroids for the ease of coil winding, with each semi-toroid accommodating about 327 turns across 9 sectors. Each sector is defined by a 2 mm deep groove to facilitate winding. For the winding, a 75-micron NbTi wire with Teflon insulation will be used.

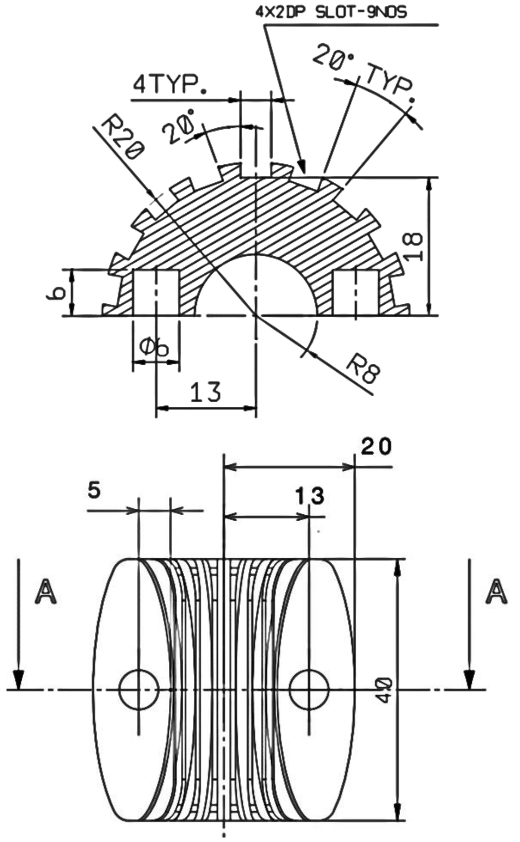

Figure 2: Engineering drawing of the PTFE core.

After winding, the two semi-toroids will be connected using a boost-fit technique, with the wire connections secured by spot welding. A test spot weld was successfully performed on the NbTi wire to ensure effective connection. The completed coil will be placed inside a copper cylindrical shield measuring 50 mm in diameter and 66 mm in length. A Teflon structure will be used to position the coil in the center of the cylinder. Both the top and bottom plates of the cylinder will also be made of copper. A subminiature type A (SMA) connector will be attached to the top plate for connecting one end of the coil, while the other end will be connected to the bottom plate. The fabrication of this resonator is currently underway.

Effective Impedance

To determine the loaded resonance frequency and calculate various trap parameters, the effective capacitance and inductance are needed. For a toroid, the total magnetic flux is the product of the number of turns, the area, and the magnetic field. If a current I flows through the coil, the magnetic field inside the toroid is $\mu_0 nI$, where n is the number of turns per unit length. Thus, the inductance of the coil can be written as $L = {\mu_0 N^2 r^2}/{2R}$. This formula applies to a toroid with a circular cross-section. However, for an elliptical cross-section, the squared radius term (r^2) can be approximated as 1/2(major axis^2+minor axis^2). In our case, this value is 212.5 mm^2. R is the mean radius of the toroid, which is 13 mm in our configuration. Using this approach, the inductance of our resonator is calculated to be 4.3 mH. This approach calculates the value of inductance that closely aligns with the inductance of the coil described in Reference [5].

To determine the capacitance of the coil, a COMSOL simulation was performed using the AC/DC module under the electrostatics (es) interface. A 2D axisymmetric simulation was conducted, with an elliptical coil placed at a radius of 13 mm, using the previously mentioned dimensions, and set at a potential of 2 V. The outer shield was maintained at zero potential. In this case plot of electric potential in color legend is shown in Fig 3. By calculating the stored electrical energy, the total capacitance in this configuration was found to be 5.93 pF. This results in an estimated resonance frequency of approximately 996 kHz for the proposed resonator. This model also predicts the capacitance for the resonator in Reference [5] to be 8 pF, giving resonance frequency of 995 kHz. This resonance frequency is closely matches with proposed resonator. Hence, even after changing the dimension of coil and shield, the predicated unloaded resonance frequency mainly depends on the length of the wire to fabricate the coil.

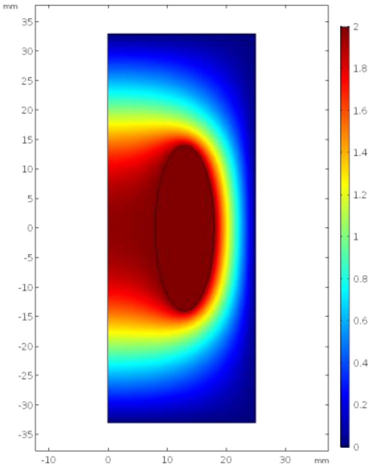

Figure 3: Plot of electric potential in color legend

In conclusion, a design of toroidal resonator has been done which will be used for the detection of light ions in cryogenic Penning trap. The effective capacitance and inductance has been determined. The unloaded resonance frequency of the resonator will be ~996 kHz.

Acknowledgement

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Reference

1. K. Blaum, Physics Reports, 425 (1) 1-78, (2006).
2. M. Block, *et. al.*, Phys. Rev. Lett., 100 132501 (2008).
3. M. Brodeur, *et. al.*, Phys. Rev. Lett., 108 52504 (2012).
4. U. Hager, *et. al.*, Phys. Rev. C **75**, . 064302 (2007).
5. C.E.Roux, Ph.D. Thesis, Ruperto-Carola-University of Heidelberg, Germany, (2012).
6. J. Nandi, *et. al.*, Rev. Sci. Inst. **93** 014706 (2022).
7. W. W. Macalpine, and R. O. Schildknecht, Phys. E: Sci. Instrum., **47** (12) 2099-2105 (1959).
8. S. Ulmer, *et.al*, Rev. Sci. Inst. **87** 113305 (2016).